Consider the canonical example, the tossing of a fair six-sided die. It can be modeled as follows:
N draws from [1, 2, 3, 4, 5, 6] (Let that be denoted by the random variable X)
The expected value is $E[X] = \frac{1+2+3+4+5+6}{6} = 3.5$
The expected sum of N draws is $E[X] \times N = 3.5N$
The standard deviation is $\sigma = \sqrt{\frac{(1-3.5)^2+...+(6-3.5)^2}{6}} = 1.71$, but that's for ONE toss!
For N tosses, the standard ERROR for the SUM (not the average!) is $\sigma\sqrt{N} = 1.71N$
But average is $\frac{TOTAL}{NUMBER}$, so the standard error for the AVERAGE is $\frac{\sigma\sqrt{N}}{N} = \frac{\sigma}{\sqrt{N}}$, which is how we got that formula
So that means, when you increase the number of draws N, you **increase** the standard error of the **sum**, but **decrease** the standard error of the **mean**. That is the key distinction.
So to maximize the chance that your **sum** is bigger than something, you want to minimize the number of standard errors it is away from the expected sum. In order to minimize the number of standard errors, you want to maximize the SIZE of the standard error because (number of standard errors from the mean)= $\frac{X-E[X]}{SE_X}$, so to minimize the # of standard errors from the mean, you want to maximize the SE_X, which you do by INCREASING N because $SE_X = \sigma\sqrt{N}$. As you can see, as you increase N, SE_X also increases, which brings down the # of standard errors you need to be off from the mean.
Get it?